

# Truth Serums for Massively Crowdsourced Evaluation Tasks

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## Abstract

A major challenge in crowdsourcing evaluation tasks like labeling objects, grading assignments in online courses, etc., is that of eliciting truthful responses from agents in the absence of verifiability. In this paper, we propose new reward mechanisms for such settings that, unlike many previously studied mechanisms, impose minimal assumptions on the structure and knowledge of the underlying generating model, can account for heterogeneity in the agents' abilities, require no extraneous elicitation from them, and furthermore allow their beliefs to be (almost) arbitrary. These mechanisms have the simple and intuitive structure of an output agreement mechanism: an agent gets a reward if her evaluation matches that of her peer, but unlike the classic output agreement mechanism, this reward is not the same across evaluations, but is inversely proportional to an appropriately defined popularity index of each evaluation. The popularity indices are computed by leveraging the existence of a large number of similar tasks, which is a typical characteristic of these settings. Experiments performed on MTurk workers demonstrate higher efficacy (with a  $p$ -value of 0.02) of these mechanisms in inducing truthful behavior compared to the state of the art.

## 1 Introduction

Systems that leverage the wisdom of the crowd are ubiquitous today. Recommendation systems such as Yelp and others, where people provide ratings and reviews for various entities, are used by millions of people across the globe [Luc11]. Commercial crowd-sourcing platforms such as Amazon Mechanical Turk, where workers perform microtasks in exchange for payments over the Internet, are employed for a variety of purposes such as collecting labelled data to train machine learning algorithms [RYZ<sup>+</sup>10]. In massive open online courses (MOOCs), students' exams or assignments are often evaluated by means of "peer-grading", where students grade each others' work [PHC<sup>+</sup>13].

A common feature in many of these applications is that they involve a large number of similar evaluation tasks and every agent performs a subset of these tasks. For instance, a typical collection of tasks on Amazon Mechanical Turk comprises of labeling a large set of images for some machine learning application. A standard Peer-grading task in massive open online courses (MOOCs) involves grading of a large number of submissions for each assignment. We call these tasks *massively crowdsourced evaluation tasks* or MCETs.

A major challenge in MCETs is incentivizing the agents to report their evaluations truthfully - to not try to game the system for monetary gain. This is achieved by designing appropriate reward

mechanisms, and there has been a considerable amount of prior work on designing such mechanisms for different settings. Unfortunately, most of these mechanisms have found limited success in practice. One critical drawback that we believe seems to impede their widespread use is the fact that these mechanisms have a complex structure and description, which makes it difficult for a typical agent to understand the mechanism and account for it while choosing their behavior. Recently, in an effort to promote practical deployments of market designs, there has been a significant push towards designing simpler economic mechanisms in the mechanism design research community [Rub15, Li15]. Indeed, simplicity in mechanism design has been a theme in many recent workshops in the research community; for instance, the abstract of one such workshop [Sim15] on “Complexity and Simplicity in Economics” quotes “*Ideal economic systems must still remain simple enough for human participants to understand...*” Due to the significant presence of the human element, these considerations are all the more important in the case of crowd-sourcing. In this paper, we attempt to address these issues in the context of MCETs by designing a class of *simple* reward mechanisms that incentivize truthful reporting.

The research on designing reward mechanisms for crowdsourced evaluation tasks falls largely into two categories depending on whether or not one assumes the existence of so-called *gold standard* objects [LEHB10, CMBN11]. These are a small subset of objects, for which the principal either knows the correct evaluations apriori or can verify them accurately. Incentive design is then facilitated by scoring the agents on their performance on these objects by using *proper scoring rules* [LS09, SZP15, SZ15].

The present paper contributes to a second line of research, that makes no assumption about the existence of such gold standard objects, and is more realistic in many applications of interest where obtaining correct evaluations for a fraction of objects is either impossible or too costly. There have been several works that operate specifically in this domain and have designed clever incentive mechanisms while making different assumptions on the behavior of the agents, and on the knowledge of the mechanism designer about this behavior [MRZ05, Pre04, WP12, RF13].

Our mechanisms build upon the structure of *output agreement mechanisms* [VAD08, VAD04] that are simple, intuitive, and have been quite popular in practice, except they suffer from a critical drawback of not incentivizing truthful responses in general. In an output agreement mechanism, two agents answer the same question, and they are both rewarded if their answers match. From the perspective of an agent, in the absence of any extraneous information, this almost incentivizes truthful reporting, since in many cases it is more likely that the other agent also has the same answer. But this is not the case when the agent believes that her answer is relatively unpopular and that a typical agent will have a different opinion. It is then tempting to report the answer that is more likely to be popular rather than correct. Moreover, there is an undesirable equilibrium in this game where every person reports the same answer irrespective of their true evaluation, which guarantees each person the highest possible payoff rewarded by the mechanism. Our mechanisms overcome these drawbacks by giving proportionately higher rewards for answers that turn out to be relatively less popular and lower rewards for answers that turn out to be more popular on an average. These rewards are designed in such a way that as soon as the agent sees the object and forms an evaluation, the conditional probabilities of the evaluations of another agent evaluating the same object change relative to the overall popularity of the different evaluations in such a way, that it becomes more profitable to report his/her opinion truthfully. This is achieved by leveraging some fundamental properties of the generating model.

We consider a standard setting that assumes the existence of an underlying generating model that captures the inherent characteristics of each of these evaluation tasks, and the abilities/biases

of the agents. The mechanisms we propose are ‘*minimal*’ in the sense that they do not solicit any extraneous information from the agents apart from their own individual evaluations. Further, they make minimal structural assumptions on the generating model and do not require the knowledge of its details. Finally, motivated by practical concerns, truthfulness is incentivized in quite a strong sense, in that, the agents are allowed to have (almost) arbitrary opinions or beliefs about the details of the generating model, e.g., one agent may grossly underestimate the abilities of the other agents and overestimate her own ability, while another agent may have no such opinions. In order to achieve these objectives, our mechanism assumes the existence of a large number of similar tasks, which is typical of MCETs.

An important distinction that naturally arises in the MCET setting is that between a *homogeneous* and a *heterogeneous* population of agents [RF15]. Homogeneity of the agents intuitively means that all agents are statistically similar in the way they answer any question; it implies, for instance, that the agents do not have any relative biases or difference in abilities. As we argue later, such an assumption is reasonable in the case of surveys, where an agent’s answer to a question can be seen as an independent sample of the distribution of the answers in the population. But it is inappropriate in subjective evaluation tasks like rating movies or grading answers, in which systematic biases may exist because of differences in preferences, effort or abilities. In our design, propose mechanisms specifically tailored to both of these settings. In the heterogeneous case it is known [RF15] that it is necessary to impose certain structural restrictions on the generating model in order to be able to design truthful mechanisms. With this in mind, we restrict ourselves to the setting of binary-choice evaluation tasks, and then propose a mechanism that is truthful under a mild *regularity* assumption that is naturally justifiable in several MCETs of interest. This assumption is substantially weaker than other assumptions that have appeared before in literature for similar settings (e.g., [DG13]).

Finally, we conduct experimental evaluation on Amazon Mechanical Turk to test how understandable or “simple” these mechanisms with an output agreement structure and popularity-scaled rewards are, and how successful they are at inducing optimal behavior. We compare one of our mechanisms with a mechanism proposed in [RF15], which is the current state of the art for the homogeneous population MCET setting, as a benchmark. The experiments reveal that our mechanism is more successful in inducing truthful behavior (with a  $p$ -value equal to 0.02).

The remainder of the paper is organized as follows. Section 2 presents a formal description of the model considered in the paper. Given the model, Section 3 puts our work in perspective of the existing literature. Section 4 and Section 5 contain the main results of the paper. Section 4 presents a mechanism to incentivize truthful reports without asking for additional information, assuming that the population is homogeneous. Section 5 then extends the results to a setting that does not make the homogeneity assumption. Our experimental results are presented in Section 6. The paper concludes with a discussion in Section 7.

## 2 Model

Consider a population denoted by the set  $\mathcal{M}$ , with  $M$  agents labelled  $j = 1, \dots, M$ . Consider an *evaluation task* in which an agent  $j$  in  $\mathcal{M}$  interacts with an object and forms an evaluation taking values in a finite set  $\mathcal{S} = (s_1, \dots, s_K)$ . Examples of object and evaluation pairs are: Movies/businesses  $\rightarrow$  ratings (e.g., Yelp, IMDB), images  $\rightarrow$  labels (in crowdsourced labeling tasks), assignments  $\rightarrow$  grades (in peer-grading). An agent’s evaluation for an object is influenced by the unknown *attributes* of the object and the manner in which these attributes affect her evaluations, or in abstract terms,

her *tastes*, *abilities* etc. Note that the attributes of an object capture everything about the object that could affect its evaluation and as such these attributes may or may not be measurable. For example, in the case where a mathematical solution is being evaluated in a peer-grading platform, its attributes could be elegance, handwriting, clarity of presentation etc., taking values: “elegance: high, handwriting: poor, clarity of presentation: poor.” Denote the hidden attribute values of an object by the quantity  $X$ , which we will simply call the *type* of the object and assume that this type takes values in a finite universe  $\mathcal{H} = \{h_1, \dots, h_L\}$ .

Denote agent  $j$ ’s evaluation for the object by  $Y_j \in \mathcal{S}$ . The manner in which an object’s attributes influence her evaluation is modeled by a conditional probability distribution over  $Y_j$  given different values of  $X$ , i.e.,  $P(Y_j = s|X = h)$  for each  $s \in \mathcal{S}$  and  $h \in \mathcal{H}$ . For notational convenience, we will denote this distribution by  $\{p_j(s|h)\}$  and we will refer to it as the “filter” of person  $j$ . Note that for each  $j$ , the filter  $\{p_j(s|h)\}$  can be represented as a stochastic matrix of size  $|\mathcal{H}| \times |\mathcal{S}|$  (recall that a stochastic matrix is one in which all the entries are non-negative and all the rows sum to 1). We assume that the filters  $\{p_j(s|h)\}$  themselves are drawn independently for each agent  $j$ , but from an identical distribution  $\mathcal{Q}$  defined on a support  $\mathcal{B}$  for all  $j$ , where  $\mathcal{B}$  is some subset of the set of all stochastic matrices of size  $|\mathcal{H}| \times |\mathcal{S}|$ .

In our setting there are  $N$  similar objects, labeled  $i = 1, \dots, N$ , that are being evaluated. The type of object  $i$  is denoted by  $X^i$  and each  $X^i$  is assumed to be drawn independently from a common probability distribution  $P_X$  over  $\mathcal{H}$ . Let  $\mathcal{M}^i \subseteq \mathcal{M}$  denote the set of persons that evaluate object  $i$  and let  $\mathcal{W}_j$  be the set of objects that a person  $j$  evaluates. If an agent  $j$  evaluates object  $i$ , let  $Y_j^i$  denote her evaluation for that object. We assume that since the objects are similar, the filters of any individual agent for evaluating the different objects are the same, i.e.,  $P(Y_j^i = s|X^i = h) = P(Y_j^{i'} = s|X^{i'} = h) = p_j(s|h)$ .

Conditional on realizations of the filters  $\{p_j(s|h)\}$  for all  $j$ , we make the following independence assumptions:

1. The evaluations  $Y_j^i$  by different  $j$  are conditionally independent given  $X^i$ .
2. The sets of random variables  $\{X^i, \{Y_j^i : j \in \mathcal{M}^i\}\}$  for the different objects  $i$  are mutually independent across objects.

In particular the second assumption implies that  $Y_j^i$  and  $Y_j^{i'}$  are independent for any person  $j$  that has evaluated objects  $i$  and  $i'$ .<sup>1</sup> Note that the random variables  $\{Y_j^i : j \in \mathcal{M}^i\}$  for a single object  $i$  need not be independent unless conditioned on  $X^i$ .

The pair of probability distribution over types  $P_X$  and the distribution  $\mathcal{Q}$  over filters is then said to comprise a *generating model* denoted as  $(P_X, \mathcal{Q})$ . In particular, given the conditional independence assumptions above, they fully specify a joint distribution on the underlying types of the different objects, the filters of the different agents, and the evaluations of the different agents of these objects.

Our goal is to design a payment mechanism that truthfully elicits evaluations from the population. With MCETs in mind, we are specifically interested in the case where  $N$  is large. The mechanism designer is not assumed to have any knowledge of  $P_X$  or the filters of the different people in the population, or of  $\mathcal{Q}$ . Further, we assume that every member of the population knows the structure of the underlying generating model, in particular the existence of a single  $P_X$  that generates the

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<sup>1</sup>This assumption precludes the possibility of dependence induced by lack of knowledge of some hidden information about an agent, e.g., if a worker’s mood is bad on a particular day, there may be a bias in all her evaluations.

type for each object, of the existence of some  $\mathcal{Q}$  that generates the filters of every member, the conditional independence assumptions on the evaluations given the type for every object, and the independence of the evaluations across different objects. But the agents may not know, or may have different subjective beliefs about the values of  $P_X$ , about  $\mathcal{Q}$ , and even their own filter. We present an example of this setting.

**Example 2.1. Peer-grading in MOOCs:** *Peer-grading, where students evaluate their peers and these evaluations are processed to assign grades to every student, has been proposed as a scalable solution to the problem of grading in MOOCs. An important component of any such scheme is the design of incentives so that students are truthful when they grade others. For example, say that the answer of any student to a fixed question has some true grade  $A$ ,  $B$  or  $C$ , which can be taken to be the type of the answer. Suppose that apriori there is a distribution over the grade of any answer that is common to all answers (to a fixed question). Each answer is then graded by a few students (and in turn each student grades a few answers), who, depending on some given rubric and their abilities, form an opinion as to what grade should be assigned to the answer. Similarly there are thousands of such answers that are graded by other students. It is natural to assume that conditional on the true grade of an answer, the evaluations of different students who grade that answer are independent. Also it is natural to assume that the grades given by the students to different answers are independent. One then wants to design a mechanism that incentivizes the students to report their true opinions about the answers that they have graded.*

Let  $q_j^i$  denote the person  $j$ 's reported evaluation for object  $i$ . Then we have the following definition of a payment mechanism.

**Definition 2.1.** *A payment (or scoring) mechanism is a set of functions  $\{\tau_j : j \in \mathcal{M}\}$ , one for each person in the population, that map the reports  $\{q_j^i : i = 1, \dots, N, j \in \mathcal{M}^i\}$  to a real valued payment (or score).*

We will work with the following notion of detail-free incentive compatibility.

**Definition 2.2.** *Consider a class  $\mathcal{C}$  of generating models. We say that a given payment mechanism  $\{\tau_j : j \in \mathcal{M}\}$  is strictly detail-free Bayes-Nash incentive compatible with respect to the class  $\mathcal{C}$  if for each  $j \in \mathcal{M}$ ,*

$$\begin{aligned} E \left[ \tau_j(\{y_j^i : i \in \mathcal{W}_j\}, \vec{Y}_{-j}) \mid \{Y_j^i = y_j^i : i \in \mathcal{W}_j\} \right] \\ > E \left[ \tau_j(\{q_j^i : i \in \mathcal{W}_j\}, \vec{Y}_{-j}) \mid \{Y_j^i = y_j^i : i \in \mathcal{W}_j\} \right], \end{aligned} \quad (1)$$

for each  $\{y_j^i : i \in \mathcal{W}_j\} \neq \{q_j^i : i \in \mathcal{W}_j\}$ , where the conditional expectation is with respect to the joint distribution on the evaluations of the population resulting from any specification of the generating model  $(P_X, \mathcal{Q})$  in class  $\mathcal{C}$ , and any  $\{p_j(s|h)\}$  in the support of  $\mathcal{Q}$ . Here  $\vec{Y}_{-j} = \{Y_{j'}^i : i = 1, \dots, N, j' \in \mathcal{M}^i, j' \neq j\}$ .

This definition implies that as long as an agent believes that the generating model is in  $\mathcal{C}$ , irrespective of whether or not she knows the generating model and her own filter, if everyone else is truthful, she gets a strictly higher payoff by being truthful. Thus truthful reporting is a strict equilibrium in the game induced by the mechanism if every agent believes that the generating model is in  $\mathcal{C}$ .

We will consider two classes of generating models inspired by two types of applications that are encountered in practice. This difference arises from the considerations for the differences in the manner in which different agents evaluate an object.

- **Homogeneous population:** Consider a typical survey, e.g., suppose the government wishes to find out the chance that a visit to the DMV office in a particular location at a particular time of the day faces a waiting time of more than 1 hour. This is a number  $X$  that can be thought of as an attribute of the DMV and for simplicity, assume that it takes values in a finite set, say  $[0, 0.1, 0.2, \dots, 1]$ . The evaluation  $Y_j$  of any agent  $j$  is just a value  $\{0, 1\}$ , with 1 denoting that she faced a wait time of greater than 2 hours. In this case it is natural to assume that  $P(Y_j = 1 | X = h) = h$ , i.e., each person's evaluation is an independent sample of the hidden value  $X$ . This means that  $p_j(s|h)$  does not depend on  $j$ , and is the same value  $p(s|h)$  for everyone. In such a case, we say that the population is homogeneous, i.e., conditioned on the type of the object, different agents form their evaluations in a statistically identical fashion. In this case,  $\mathcal{Q}$  has its support on a single filter: the population filter  $\{p(s|h)\}$ . We will consider this case in Section 4.
- **Heterogeneous population:** In most subjective evaluations, the manner in which agents form evaluations differ considerably due to differences in preferences, abilities etc. So it is natural to assume that the filters vary across the population, i.e.  $\mathcal{Q}$  has a support of size larger than 1. We will consider this case in Section 5. In this case, in general it is impossible to design detail-free truthful mechanisms for this case unless some additional structural assumptions are made on the support of  $\mathcal{Q}$ . We will propose a natural structural assumption for the case  $|\mathcal{S}| = 2$ , i.e., in the case where the evaluations are binary, and design a truthful mechanism under this assumption.

### 3 Related work

The theory of elicitation of private evaluations or predictions of events has a rich history. In the standard setting, an agent possesses some private information in the form of an evaluation of some object or some informed prediction about an event, and one would like to elicit this private information. There are two categories of these problems. In the first category, the ground truth, e.g., true quality or nature of the object or the knowledge of the realization of the event that one wants to predict, is available or will be available at a later stage. In this case, the standard technique is to score an agent's reports against the ground truth, and proper scoring rules [GR07, Sav71, LS09] provide an elegant framework to do so. In the second category of problems, the ground truth is not known. In this case there is little to be done except to score these reports against the reports of other agents who have provided similar predictions about the same event. The situation is then inherently strategic, in which one hopes to sustain truthful reporting as an equilibrium of a game: assuming all the other agents provide their predictions truthfully, these predictions form an informative ensemble, and with a carefully designed rule that scores reports against this ensemble, one incentivizes any agent to also be truthful. The present work falls in this category.

In this category, majority of early literature has focused on the case where a single object is being evaluated. In a pioneering work, the peer-prediction method by [MRZ05] assumed that the population is homogeneous and the mechanism designer knows the agents' beliefs about the underlying generating model of evaluations. In this case they demonstrated the use of proper-scoring rules to design a truthful mechanism that utilizes the knowledge of these subjective beliefs.

These mechanisms are minimal in the sense that they only require agents to report their evaluations. In another influential work, [Pre04] considered a homogeneous population and designed an *oblivious* mechanism, famously termed Bayesian truth serum (BTS), that does not require the knowledge of the underlying generating model, but requires that the number of agents is large and that they have a common prior, i.e., they have the same beliefs about the underlying generating model and this fact is common knowledge. This mechanism is not minimal: apart from reporting their evaluations, agents are also required to report their beliefs about the reports of others. [WP12] and [RF13] later used proper-scoring rules to design similar mechanisms for the case where the population size is finite. These mechanisms are again not minimal, and in fact it is known (see [JF11], [RF13]) that no minimal mechanism that does not use the knowledge of the prior beliefs can incentivize truthful reporting of evaluations.

It is the case in many applications in crowd-sourcing, that one is interested in acquiring evaluations from a population for several similar objects. It is thus natural to explore the possibility of exploiting this statistical similarity to design better (e.g. minimal) mechanisms for jointly scoring these evaluation tasks. This is the context of the present work. Three major works in this area that have considered this case are [WP13], [DG13] and more recently, [RF15]. Both [WP13] and [DG13] only considered the case where the evaluations are binary. The former considered a homogeneous population while the latter considered a heterogeneous population, while both making specific assumptions on the generating model. [RF15] on the other hand have considered both homogeneous and heterogeneous populations.

For a homogeneous population with multiple objects, [WP13] try to utilize the statistical independence of the objects to estimate the prior distribution of evaluations and use that to compute payments using a proper scoring rule. In spirit, we are similar to this approach (and also [Pre04]) in the sense that we use the law of large numbers to estimate some prior statistics and we get incentive compatibility for a large population, but we do not restrict ourselves to the binary setting. [RF15] recently have also designed a mechanism that is truthful in the general non-binary setting while requiring only a finite number of objects, again using proper scoring rules. In their mechanism, for computing the reward to an agent for evaluating a given object, a sample of evaluations of other agents for other objects of a fixed size needs to be collected, and an agent's reward can be non-zero only if this sample is sufficiently rich, i.e., it has an adequate representation of all the possible evaluations. Although our mechanism needs the number of objects to be large, it has a much simpler structure.

For the case of heterogeneous population, [RF15] show that typically one cannot guarantee truthfulness with minimal elicitation. Nevertheless, [DG13] have designed truthful minimal mechanism for the case of binary evaluations for a specific generating model: it is assumed that  $\mathcal{H} = \mathcal{S}$  and for each agent, the probability of correctly guessing the true type of the object is at least 0.5 and it does not depend on the type. Although we also consider the binary evaluations, we allow  $\mathcal{H}$  to be arbitrary and our regularity condition is considerably weaker.

In a parallel development, [SAFP16] elegantly extended the mechanism in [DG13] for the heterogeneous population setting to handle more than two evaluations. But their mechanism is truthful only if the joint distribution of the evaluations seen by two agents evaluating the same object satisfy a property that they refer to as being "categorical". It is a somewhat restrictive condition which says that if an agent makes an evaluation  $s_1$ , then the conditional probability that the other agent makes a certain other evaluation  $s_2$  reduces relative to the prior probability of making that evaluation, for every other evaluation  $s_2$ . If this condition is satisfied in our setting, then an "additive" mechanism that we suggest for the case of a heterogeneous population is trivially truthful for non-

binary settings, almost by assumption. They also design a mechanism that is truthful in general, in particular without this restriction, but they require that the mechanism designer has access to certain information about the joint distribution of the evaluations for an object by two agents. In another parallel development [RFJ16], the authors consider the heterogeneous population setting and propose almost exactly the same mechanism as ours: it has an output agreement structure where the rewards for matching on an evaluation are inversely proportional to an estimate of the prior probability of seeing that evaluation. They show that the mechanism is truthful for the general non-binary setting, but the condition under which this holds is the same as the condition for truthfulness, and it is not clear if it would reasonably hold in practical settings.

Contrary to the assumptions in these two works our regularity condition is a precise condition on the generating model that can be mapped to a condition on the "behavior" of the agents. It implies both the condition in [RFJ16] and the condition in [SAFP16] in the binary setting, and further it can be argued to naturally hold in most MCETs of interest.

## 4 Homogeneous population

In this section, we will first consider the case where the population of agents is homogeneous. We will consider the following class of generating models  $(P_X, \mathcal{Q})$ , that we will call  $\mathcal{C}_{hom}$ .

**Definition 4.1.**  $\mathcal{C}_{hom}$  is the class of all generating models  $(P_X, \mathcal{Q})$  that satisfy the following set of assumptions.

1. The population is homogeneous, i.e.,  $\{p_j(s|h)\} = \{p_{j'}(s|h)\}$  for any  $j, j' \in \mathcal{M}$ . This means that  $\mathcal{Q}$  has support of size 1.  $\{p(s|h)\}$  denotes the common population filter.
2. Define

$$\delta(P_X, \mathcal{Q}) = \min_{s_k, s_l \in \mathcal{S}, s_k \neq s_l} \sqrt{\left(\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)^2\right) \left(\sum_{h \in \mathcal{H}} P_X(h) p(s_l|h)^2\right) - \sum_{h \in \mathcal{H}} P_X(h) p(s_k|h) p(s_l|h)}.$$

By the Cauchy-Schwarz inequality  $\delta(P_X, \mathcal{Q}) \geq 0$ . Then there is some  $\delta_0 > 0$  such that

$$\inf_{(P_X, \mathcal{Q}) \in \mathcal{C}_{hom}} \delta(P_X, \mathcal{Q}) > \delta_0. \quad (2)$$

Let us take a closer look at the second assumption. The Cauchy-Schwarz inequality has the following geometric interpretation. For any evaluation  $s \in \mathcal{S}$ , define the vector

$$v(s) \triangleq [\sqrt{P_X(h_1)} p(s|h_1), \dots, \sqrt{P_X(h_L)} p(s|h_L)], \quad (3)$$

in the Euclidean space  $\mathbb{R}^L$ . Then the Cauchy-Schwarz inequality says that for any two evaluations  $s_k$  and  $s_l$ , the magnitude of the projection of the vector  $v(s_k)$  on the unit vector in the direction  $v(s_l)$  is less than the magnitude of the vector  $v(s_k)$  itself (one can reverse the roles of  $s_k$  and  $s_l$ ), i.e.,

$$\frac{|v(s_k) \cdot v(s_l)|}{\|v(s_l)\|} \leq \|v(s_k)\|,$$

or

$$|v(s_k) \cdot v(s_l)| \leq \|v(s_k)\| \|v(s_l)\|.$$



If we let  $\theta(v(s_k), v(s_l))$  denote the angle in radians between two non-zero vectors  $v(s_k)$  and  $v(s_l)$ , defined as

$$\theta(v(s_k), v(s_l)) \triangleq \arccos \frac{v(s_k) \cdot v(s_l)}{\|v(s_k)\| \|v(s_l)\|}, \quad (4)$$

then the inequality is strict if and only if the angle between the vectors  $v(s_k)$  and  $v(s_l)$  is positive and their magnitude is non-zero. In fact, under the condition that  $\|v(s)\| \leq 1$  for all  $s \in \mathcal{S}$ , which holds in our case, we can show that the second assumption is equivalent to the following assumption.

**Assumption A:** There is a  $\tau_0 > 0$  and  $\kappa_0 > 0$  such that for any  $(P_X, \mathcal{Q}) \in \mathcal{C}_{hom}$ , the following holds:

1.  $\sum_{h \in \mathcal{H}} P_X(h) p(s|h) > \tau_0$  for each  $s \in \mathcal{S}$ , and
2.  $\theta(v(s_k), v(s_l)) > \kappa_0$  (note that since these  $v(s)$  are component-wise positive, we have  $\kappa_0 \leq \pi/2$ ).

The first condition says that the probability of an agent forming any evaluation  $s \in \mathcal{S}$  for an object is uniformly bounded away from zero for all generating models in the class. To get an intuition for the second condition, consider the case the angle between  $v(s_k)$  and  $v(s_l)$  is zero. One can show that this happens only when there is a  $C \in \mathbb{R}$  such that  $p(s_k|h) = Cp(s_l|h)$  for each  $h \in \mathcal{H}$  such that  $P_X(h) > 0$ . But this case the evaluations  $s_k$  and  $s_l$  need not be distinguished at all, since they contain the same information about  $X^i$ . In particular,  $P(X^i = h | Y_j^i = s_k) = P(X^i = h | Y_j^i = s_l)$  for each  $h \in \mathcal{H}$ . Hence one can equivalently consider a generating model with a fewer number of possible evaluations.

**Proposition 1.** *Assumption A and assumption 2 in definition 4.1 are equivalent.*

*Proof.* To see that assumption A implies the second assumption, note that  $\theta(v(s_k), v(s_l)) \geq \kappa_0$  implies that:

$$\frac{v(s_k) \cdot v(s_l)}{\|v(s_k)\| \|v(s_l)\|} \leq \cos \kappa_0,$$

Multiplying throughout by  $\|v(s_k)\| \|v(s_l)\|$ , we have:

$$\|v(s_k)\| \|v(s_l)\| - v(s_k) \cdot v(s_l) \geq (1 - \cos \kappa_0) \|v(s_k)\| \|v(s_l)\| \geq (1 - \cos \kappa_0) \tau_0^2 > 0.$$

Here in the last inequality, we use that fact that

$$\|v(s)\| = \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2} \geq \sum_{h \in \mathcal{H}} P_X(h) p(s|h) > \tau_0,$$

which follows from the Jensen's inequality. The reverse direction is less straightforward and this is where we need to use the fact that  $\|v(s)\| \leq 1$  for all  $s$ . First of all

$$|v(s_k) \cdot v(s_l)| \leq \|v(s_k)\| \|v(s_l)\| - \delta_0,$$

implies that either  $\|v(s_k)\|$  or  $\|v(s_l)\|$  is non-zero. Say  $\|v(s_l)\| > 0$ . Then dividing on both sides, we get:

$$\begin{aligned} \frac{|v(s_k) \cdot v(s_l)|}{\|v(s_l)\|} &\leq \|v(s_k)\| - \frac{\delta_0}{\|v(s_l)\|} \\ &\leq \|v(s_k)\| - \delta_0 \end{aligned}$$

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**Mechanism 1:** Hom-OA for a homogeneous population. Assumes that  $|\mathcal{M}^i| \geq 3$ .

---

The observations of all the people for the different objects are solicited. Let these be denoted by  $\{q_j^i\}$ , where every  $q_j^i \in \mathcal{S}$ . A person  $j$ 's payment is computed as follows:

- From each population  $\mathcal{M}^i$ , choose any two persons  $j_1$  and  $j_2$  different from  $j$ , and for each possible evaluation  $s \in \mathcal{S}$ , compute the quantity

$$f_j^i(s) = \mathbf{1}_{\{q_{j_1}^i=s\}} \mathbf{1}_{\{q_{j_2}^i=s\}}$$

Then compute

$$\bar{f}_j(s) = \frac{1}{N} \sum_{i=1}^N f_j^i(s).$$

- For each evaluation  $s$ , fix a payment  $r_j(s)$  defined as

$$r_j(s) = \begin{cases} \frac{K}{\sqrt{\bar{f}_j(s)}} & \text{if } \bar{f}_j(s) \neq 0, \\ 0 & \text{if } \bar{f}_j(s) = 0, \end{cases}$$

where  $K > 0$  is any positive constant.

- For computing person  $j$ 's payment for evaluating object  $o \in \mathcal{W}_j$ , choose another person  $j'$  who has evaluated the same object  $o$ . If their reports match, i.e., if  $q_j^i = q_{j'}^i = s$ , then the person  $j$  gets a reward of  $r_j(s)$ . If the reports do not match, then  $j$  gets 0 payment for the evaluation of that object.
- 

where the last inequality holds since  $\|v(s_k)\| \leq 1$ . In other words:

$$\|v(s_k)\| \cos \theta(v(s_k), v(s_l)) \leq \|v(s_k)\| - \delta_0,$$

or

$$\|v(s_k)\| (1 - \cos \theta(v(s_k), v(s_l))) \geq \delta_0.$$

Since  $\|v(s_k)\| \leq 1$  and  $(1 - \cos \theta(v(s_k), v(s_l))) \in [0, 1]$ , this implies both  $\|v(s_k)\| \geq \delta_0$  and  $(1 - \cos \theta(v(s_k), v(s_l))) \geq \delta_0$ , i.e.,  $\theta(v(s_k), v(s_l)) \geq \arccos(1 - \delta_0)$ . Finally we have  $\sum_{h \in \mathcal{H}} P_X(h) p(s|h) \geq \|v(s_k)\|^2 \geq \delta_0^2$ . Note that  $\delta_0 \leq 1$  so that  $\arccos(1 - \delta_0) \leq \pi/2$ .

□

We present our proposed mechanism for this case in Mechanism 1, denoted as Hom-OA, where OA stands for output agreement. The mechanism has the structure of an output agreement mechanism, where a person is rewarded for evaluated an object only if his evaluation matches that of a chosen peer who has evaluated the same object. In our mechanism this reward itself depends on how "popular" the matched evaluation is overall across all the objects, where the notion of popularity is defined in a particular manner. In the following theorem, we show that the mechanism is Bayes-Nash incentive compatible for a large enough  $N$ .

**Theorem 2.** *There exists a  $N_0$  depending only on  $\delta_0$  and  $K$  such that if the number of objects is  $N > N_0$ , Mechanism 1 is strictly detail-free Bayes-Nash incentive compatible with respect to the class  $\mathcal{C}_{hom}$ .*

*Proof.* First, note that the computation of the payments  $r_j(s_k)$  for the different  $s_k$  is unaffected by the reports of person  $j$ . Next, suppose that everyone but a person  $j$  is truthful. Recalling the definition of  $v(s)$ , denote

$$E(\bar{f}_j(s)) = E\left(\frac{1}{N} \sum_{i=1}^N f_j^i(s)\right) = \sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2 = \|v(s)\|^2 \triangleq g(s).$$

In the proof of Proposition 1, we have seen that the second assumption in the definition of class  $\mathcal{C}_{hom}$  implies that  $\|v(s)\| > \delta_0$ , and thus we have  $g(s) > \delta_0^2 > 0$  for all  $s \in \mathcal{S}$ . Next, recall that

$$r_j(s) = \mathbf{1}_{\bar{f}_j(s) \neq 0} \frac{K}{\sqrt{\bar{f}_j(s)}}.$$

We will show first that

$$E(r_j(s)) = \frac{K}{\sqrt{g(s)}} + m(N),$$

where there exists a function of  $N$ ,  $\sigma(N) \geq 0$ , that depends only on  $K$  and  $\delta_0$ , i.e., it is independent of the generating model, such that  $|m(N)| \leq \sigma(N)$  and  $\sigma(N) = o(1)$ . i.e.,  $\lim_{N \rightarrow \infty} \sigma(N) = 0$ . To show this, we first have for any  $\epsilon > 0$ :

$$\begin{aligned} E(r_j(s)) &\geq P(\bar{f}_j(s) \in [g(s)(1-\epsilon), g(s)(1+\epsilon)]) \frac{K}{\sqrt{g(s)(1+\epsilon)}} \\ &\geq (1 - 2\exp(-\epsilon^2 g(s)^2 N)) \frac{K}{\sqrt{g(s)(1+\epsilon)}} \\ &\geq (1 - 2\exp(-\epsilon^2 \delta_0^4 N)) \frac{K}{\sqrt{g(s)(1+\epsilon)}} \\ &\geq \frac{K}{\sqrt{g(s)(1+\epsilon)}} - 2\exp(-\epsilon^2 \delta_0^4 N) \frac{K}{\delta_0 \sqrt{(1+\epsilon)}} \\ &\geq \frac{K}{\sqrt{g(s)}} \left(1 - \frac{\epsilon}{2} + h(\epsilon)\right) - 2\exp(-\epsilon^2 \delta_0^4 N) \frac{K}{\tau_0 \sqrt{(1+\epsilon)}}. \end{aligned}$$

Here the second inequality follows from Hoeffding's inequality, and the fifth is the Taylor series approximation of the function  $1/\sqrt{1+\epsilon}$ , where  $h(\epsilon) = o(\epsilon)$ . The other inequalities result from the fact that  $g(s) \geq \delta_0^2$ . Thus we have

$$E(r_j(s)) \geq \frac{K}{\sqrt{g(s)}} - \frac{\epsilon K}{2\delta_0} - \frac{K|h(\epsilon)|}{\delta_0} - 2\exp(-\epsilon^2 \delta_0^4 N) \frac{K}{\delta_0 \sqrt{(1+\epsilon)}}$$

Taking  $\epsilon = N^{-1/4}$ , we have:

$$E(r_j(s)) \geq \frac{K}{\sqrt{g(s)}} - l(N) \tag{5}$$

where  $l(N) \geq 0$  and as a function of  $N$ , it depends only on  $\delta_0$  and  $K$ , and further  $\lim_{N \rightarrow \infty} l(N) = 0$ .

Next we also have,

$$\begin{aligned}
E(r_j(s)) &\leq P(\bar{f}_j(s) \in [g(s)(1-\epsilon), g(s)(1+\epsilon)]) \frac{K}{\sqrt{g(s)(1-\epsilon)}} \\
&\quad + E\left(\mathbf{1}_{\bar{f}_j(s) \notin \{0\} \cup [g(s)(1-\epsilon), g(s)(1+\epsilon)]} \frac{K}{\bar{f}_j(s)}\right) \\
&\leq \frac{K}{\sqrt{g(s)(1-\epsilon)}} + P\left(\mathbf{1}_{\bar{f}_j(s) \notin \{0\} \cup [g(s)(1-\epsilon), g(s)(1+\epsilon)]}\right) KN \\
&\leq \frac{K}{\sqrt{g(s)(1-\epsilon)}} + 2KN \exp(-\epsilon^2 g(s)^2 N) \\
&\leq \frac{K}{\sqrt{g(s)(1-\epsilon)}} + 2KN \exp(-\epsilon^2 \delta_0^4 N) \\
&\leq \frac{K}{\sqrt{g(s)}} \left(1 + \frac{\epsilon}{2} + w(\epsilon)\right) + 2KN \exp(-\epsilon^2 \delta_0^4 N) \\
&\leq \frac{K}{\sqrt{g(s)}} + \frac{\epsilon K}{2\delta_0} + \frac{|w(\epsilon)|K}{\delta_0} + 2KN \exp(-\epsilon^2 \delta_0^4 N)
\end{aligned}$$

Here the second inequality results from the fact that on the event  $\{\bar{f}_j(s) \neq 0\}$ ,  $\bar{f}_j(s) \geq 1/N$ . This is because  $\bar{f}_j(s)$  only takes values in the set  $[0, \frac{1}{N}, \frac{2}{N}, \dots, 1]$ . The third inequality follows from Hoeffding's inequality, and the fifth follows from the Taylor approximation of the function  $1/\sqrt{1-\epsilon}$ , where  $w(\epsilon) = o(\epsilon)$ . Now choosing  $\epsilon = N^{-1/4}$ , we get:

$$E(r_j(s)) \leq \frac{K}{\sqrt{g(s)}} + u(N) \quad (6)$$

where  $u(N) \geq 0$  and as a function of  $N$ , it depends only on  $\delta_0$  and  $K$ . Further  $\lim_{N \rightarrow \infty} u(N) = 0$ . Hence defining  $\sigma(N) = \max(u(N), l(N))$  we have  $|m(N)| \leq \sigma(N)$  where  $\sigma(N) = o(1)$  and it depends on  $\delta_0$  and  $K$ .

The expected reward of person  $j$  for evaluating object  $i$ , if she reports  $q_j^i = s_l$  when her true evaluation is  $s_k$  is

$$R(s_l, s_k) = P(Y_{j'}^i = s_l | Y_j^i = s_k) E(r_j(s_l)) = \frac{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h) p(s_l|h)}{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)} E(r_j(s_l))$$

Similarly, we have  $R(s_k, s_k) = \frac{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)^2}{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)} E(r_j(s_k))$ . Next, lying is strictly worse for person  $j$  if  $R(s_l, s_k) < R(s_k, s_k)$ , that is if

$$\begin{aligned}
&\frac{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h) p(s_l|h)}{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)} \left( \frac{K}{\sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s_l|h)^2}} + m(N) \right) \\
&< \frac{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)^2}{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)} \left( \frac{K}{\sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)^2}} + m(N) \right),
\end{aligned}$$

i.e., if

$$\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h) p(s_l|h) < \sqrt{\left(\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)^2\right) \left(\sum_{h \in \mathcal{H}} P_X(h) p(s_l|h)^2\right)} - \frac{|m(N)|}{K}.$$

Further, for every generating model in class  $\mathcal{C}_{hom}$ , we have

$$\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h) p(s_l|h) + \delta_0 < \sqrt{\left(\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)^2\right) \left(\sum_{h \in \mathcal{H}} P_X(h) p(s_l|h)^2\right)}.$$

where  $\delta_0 > 0$ . Thus truthtelling gives a strictly better payoff if  $\frac{|m(N)|}{K} < \delta_0$ . Since  $|m(N)| \leq \sigma(N)$ , which depends only on  $\delta_0$  and  $K$  and  $\sigma(N) = o(1)$ , there is an  $N_0$  depending only on  $\delta_0$  and  $K$  such that for all  $N > N_0$ ,  $\frac{|m(N)|}{K} < \delta_0$  irrespective of the generating model in  $\mathcal{C}_{hom}$ .  $\square$

Note how detail-freeness follows from the fact that the proof does not depend on the specific filter  $\{p(s|h)\}$ , but depends on a universal property shared by any such filter, i.e, the Cauchy-Schwarz inequality.

#### 4.1 Remarks:

**An alternative to the peer prediction method:** In the case where the mechanism designer knows the underlying generating model  $P_X$  and  $\{p(s|h)\}$ , the mechanism can compute the rewards for each evaluation directly, without having to estimate statistics from evaluations for multiple objects. In order to do so, for each evaluation  $s_k$ , one defines

$$g(s_k) = \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s_k|h)^2},$$

and defines payments  $r(s_k)$  for the different evaluations as  $r(s_k) = \frac{K}{g(s_k)}$  (in case  $g(s_k) = 0$  for some  $s_k$ , then one can simply not allow that signal to be reported). In this case, our mechanism provides an alternative to the peer prediction method of [MRZ05], while using the simple structure of output agreement mechanisms and without using proper scoring rules.

**Relaxing the requirement that  $|\mathcal{M}_i| \geq 3$ :** The assumption  $|\mathcal{M}_i| \geq 3$  is needed to ensure that for any object, there are at least two persons other than any given person  $j$  who have evaluated the object, which in turn ensures that the computation of the  $\bar{f}_j(s)$  values are unaffected by the reports of  $j$ . In practice, even if one computes the  $\bar{f}_j(s)$  values by randomly selecting any two persons for each object, as long as  $|\mathcal{W}_j|$  is small and  $N$  is large, these values will not be affected much by the reports of  $j$ . In this case, one can drop the subscript  $j$ , and use the same values  $\bar{f}(s)$  to compute everyone's payment.

**Truthfulness gives higher payoff than random sampling:** Truthful reporting is not the only equilibrium of this mechanism. One class of equilibria is where each person in the population reports evaluations sampled independently from the same distribution, say  $\{z(s) : s \in \mathcal{S}\}$ , independent of their observations. But one can easily show that the truthful equilibrium gives more reward in expectation to each person than in any of the equilibria in this class. In this case, we have  $E(r_j(s)) = \frac{K}{z(s)} + m(N)$  for all  $s$  such that  $z(s) > 0$ , where  $m(N) = o(1)$ . Thus the expected payment of each agent for evaluation of one object is  $\sum_{s \in \mathcal{S}} z(s)^2 \left(\frac{K}{z(s)} + m(N)\right) = K + r(N)$ , where  $r(N) = o(1)$  whereas the expected payment for evaluation of one object in the truthful equilibrium is

$$\sum_{s \in \mathcal{S}} P(Y_j = s) P(Y_{j'} = s | Y_j = s) E(r_j(s)) = \sum_{s \in \mathcal{S}} \frac{K \sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2}{\sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2}} + q(N)$$

$$= K \sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2} + q(N) \geq K \sum_{s \in \mathcal{S}} P(Y_j = s) + q(N) = K + q(N),$$

where  $q(N) = o(1)$ . The last inequality follows from the Jensen's inequality. In fact, this inequality is strict for the class  $\mathcal{C}_{hom}$  with a gap that is universally bounded away from zero. To see this, note that the inequality is not strict only when  $Y_j$  and  $X$  are independent, i.e., the population filter is such that the evaluations are independent of the type of the object. But if that is the case, one can verify that  $\delta((P_X, \mathcal{Q})) = 0$ <sup>2</sup>, thus violating our assumption. Thus for a large enough  $N$ , the truthful equilibrium gives a strictly higher expected payoff.

**Conjecture - Truthfulness gives higher payoff than any symmetric equilibrium:** We conjecture that for a large enough  $N$ , truthful reporting gives at least as much payoff to each individual as in any symmetric equilibrium, where each person in the population maps the observed evaluation  $s$  for any object to a reported evaluation  $s'$  with some probability  $q(s'|s)$  for each  $s, s' \in \mathcal{S}$ . In fact, we conjecture that under the conditions satisfied by  $\mathcal{C}_{hom}$ , the payoff is strictly higher compared to all symmetric equilibria except the ones that result from relabelling the signals. Any symmetric equilibrium is equivalent to the truthful equilibrium in which the population filter is given by:

$$p'(s|h) = \sum_{s' \in \mathcal{S}} p(s'|h) q(s|s').$$

The expected payment of each agent for evaluation of one object in the truthful equilibrium is

$$\begin{aligned} \sum_{s \in \mathcal{S}} P(Y_j = s) P(Y_{j'} = s | Y_j = s) E(r_j(s)) &= \sum_{s \in \mathcal{S}} \frac{K \sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2}{\sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2}} + q(N) \\ &= K \sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2} + q(N) \end{aligned}$$

where  $q(N) = o(1)$ . Similarly, the expected payoff in any symmetric equilibrium is:

$$K \sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p'(s|h)^2} + r(N) = K \sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) \left( \sum_{s' \in \mathcal{S}} p(s'|h) q(s|s') \right)^2} + r(N)$$

We conjecture that the following inequality holds in general, and strictly under the assumptions of the class  $\mathcal{C}_{hom}$ :

$$\sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2} \geq \sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) \left( \sum_{s' \in \mathcal{S}} p(s'|h) q(s|s') \right)^2}. \quad (7)$$

We have not been able to prove or disprove it thus far. The inequality has the following interpretation. Consider the following definition.

**Definition 4.2.** Consider two random variables  $Y_1$  and  $Y_2$ , taking values in a finite set  $\mathcal{S}$ , such that they are conditionally independent and identically distributed given some random variable  $X$  taking values in a finite set  $\mathcal{H}$ . Then the agreement measure between  $Y_1$  and  $Y_2$  is defined as

$$\Gamma(Y_1, Y_2) = \sum_{s \in \mathcal{S}} \sqrt{P(Y_1 = Y_2 = s)}$$

<sup>2</sup>To see this, observe that for any  $s_k$  and  $s_l$ , the angle between the two vectors  $v(s_k) = P(Y_j = s_k)[\sqrt{P_X(h_1)}, \dots, \sqrt{P_X(h_L)}]$  and  $v(s_l) = P(Y_j = s_l)[\sqrt{P_X(h_1)}, \dots, \sqrt{P_X(h_L)}]$  is 0, and hence the Cauchy-Schwarz inequality is not strict.

Now if  $X$  has distribution  $P_X$ , and the conditional distributions of  $Y_1$  and  $Y_2$  given  $X$  are denoted as  $\{p(s|h)\}$ , then the agreement measure is  $\sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2}$ , and this is the expected payoff to an individual under the truthful equilibrium in our mechanism. The agreement measure has the following properties:

1.  $\Gamma(Y_1, Y_2) \geq 1$ . To see this, note that Jensen's inequality implies that

$$\sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2} \geq \sum_{s \in \mathcal{S}} \sum_{h \in \mathcal{H}} P_X(h) p(s|h) = 1.$$

In fact  $\Gamma(Y_1, Y_2) = 1$  only when  $Y_1$  and  $Y_2$  are independent.

2.  $\Gamma(Y_1, Y_2) \leq \sqrt{|\mathcal{S}|}$ . To see this, note that Jensen's inequality implies that

$$\begin{aligned} \sum_{s \in \mathcal{S}} \sqrt{\sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2} &\leq |\mathcal{S}| \sqrt{\frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \sum_{h \in \mathcal{H}} P_X(h) p(s|h)^2} \\ &\leq |\mathcal{S}| \sqrt{\frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \sum_{h \in \mathcal{H}} P_X(h) p(s|h)} = \sqrt{|\mathcal{S}|}. \end{aligned}$$

In fact  $\Gamma(Y_1, Y_2) = \sqrt{|\mathcal{S}|}$  only when  $Y_1$  and  $Y_2$  are identical and they are distributed uniformly, i.e.,  $Y_1 = Y_2$  and  $P(Y_1 = s) = 1/|\mathcal{S}|$ .

Now our conjecture is true if the agreement measure has the following property. Suppose  $Z_1$  and  $Z_2$  are two random variables such that 1)  $Z_1$  and  $X$  are conditionally independent given  $Y_1$ , and  $Z_2$  and  $X$  are conditionally independent given  $Y_2$  and 2)  $Z_1$  and  $Z_2$  have the same conditional distributions given  $Y_1$  and  $Y_2$  respectively. Then clearly  $Z_1$  and  $Z_2$  are conditionally independent and identically distributed given  $X$ . Then we would like to show that

$$\Gamma(Y_1, Y_2) \geq \Gamma(Z_1, Z_2).$$

That is, intuitively, the agreement measure decreases if some information in  $Y_1$  and  $Y_2$  is unilaterally lost.

## 5 Heterogeneous population

We now consider the case of a heterogeneous population. In this case it is known (see [RF15]) that unless additional structural assumptions are made on the class of generating models, it can be impossible to design a strictly detail-free Bayes-Nash incentive compatible mechanism. To get an intuition for the key issue, consider the following example. Assume a movie is being evaluated. Suppose that its type is in the set  $\mathcal{H} = \{Action, Drama\}$  and that the set of evaluations is  $\mathcal{S} = \{Good, Bad\}$ . Consider two filters: an ‘action-lover’ filter and a ‘drama-lover’ as shown in the Figure 1. Now consider a generating model such that both these filters are in the support of  $\mathcal{Q}$  and consider an agent, say Bob, that has an action-lover filter and another agent *Alice* who has a drama-lover filter. Now observe that the filters are such that the conditional distribution over the evaluations of all the other agents in the population from the point of view of Bob, given that he evaluates the movie to be good, is the same as the conditional distribution over the evaluations of all the other agents in the population from the point of view of Alice, given that she evaluates the

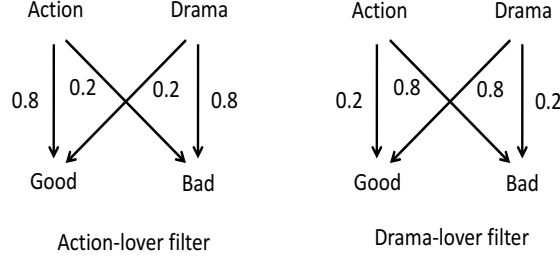


Figure 1: Two filters.

movie to be bad. So any strictly detail-free Bayes-Nash incentive compatible mechanism that strictly incentivizes Bob to report ‘Good’ when his evaluation is ‘Good’ also simultaneously incentivizes Alice to report ‘Good’ when her evaluation for the movie is ‘Bad’. Hence there can be no mechanism that is strictly Bayes-Nash incentive compatible with respect to this generating model.

### 5.1 Regular filters

The previous result intuitively implies that if the preferences of the agents in the population varies too much, then designing a knowledge independent strictly truthful mechanism is impossible. One thus needs to impose some uniformity or *regularity* on the population filters for there to be any hope of designing such mechanisms. With this in mind, we look at the specific case of binary evaluations, i.e.,  $|\mathcal{S}| = 2$ , and impose a notion a regularity on the filters.

**Definition 5.1.** *We say that a generating model  $(P_X, \mathcal{Q})$  with  $|\mathcal{S}| = 2$  is  $\delta$ -regular for some  $\delta \geq 0$  if there is a fixed ordering of types in the set  $\mathcal{H}$ , say,*

$$h_1 \succ h_2 \succ \dots \succ h_L,$$

*such that every filter  $\{p_j(s|h)\}$  in the support of  $\mathcal{Q}$  satisfies the following regularity condition with respect to this ordering:*

$$p_j(s_1|h) > p_j(s_1|h') + \delta \quad \text{if} \quad h \succ h' \quad (8)$$

*for each  $h \neq h' \in \mathcal{H}$ . Note that this implies that  $p_j(s_2|h) < p_j(s_2|h') - \delta$  if  $h \succ h'$ .*

It follows from the definition that if a generating model is  $\delta$ -regular, then it is also  $\delta'$ -regular for any  $\delta' \in [0, \delta]$ . Intuitively, the regularity condition implies that the types that are higher in the ordering are in some sense "closer" to  $s_1$  and those lower in the ordering are "farther away" from  $s_1$ , and vice versa for  $s_2$ , so that an agent making a particular evaluation is more likely if the object type is "closer" to that evaluation. Note that for any particular filter  $p_j(\cdot|\cdot)$ , as long as  $p(s_1|h) \neq p(s_1|h')$  for any  $h, h'$ , one can always define an ordering of the types such that this condition will be satisfied for that filter (if this is not satisfied for some  $h$  and  $h'$ , then  $h$  and  $h'$  can be clustered to form a single type). But the regularity condition says that there is one such fixed ordering of types for *all* filters in the support of  $\mathcal{Q}$ .

**Definition 5.2.** *For a generating model  $(P_X, \mathcal{Q})$ , the ensemble filter is defined as*

$$p(s|h) \triangleq E_{\mathcal{Q}}(p_j(s|h)),$$

*where the expectation is with respect to  $\mathcal{Q}$ .*



Clearly, if a generating model is  $\delta$ -regular, then the ensemble filter satisfies the regularity condition as well.

Let us take a closer look at the  $\delta$ -regularity assumption with the example of peer grading. Consider the case where  $|\mathcal{H}| = |\mathcal{S}| = 2$ . In this case, w.l.o.g., we can assume that  $\mathcal{H} = \mathcal{S} = \{h_1, h_2\}$ . Then the  $\delta$ -regularity condition will be satisfied if either:

- $p_j(h_1|h_1) > p_j(h_1|h_2) + \delta$  (and thus  $p_j(h_2|h_2) > p_j(h_2|h_1) + \delta$ ) for all filters in support of  $\mathcal{Q}$ .
- $p_j(h_1|h_2) > p_j(h_1|h_1) + \delta$  (and thus  $p_j(h_2|h_1) > p_j(h_2|h_2) + \delta$ ) for all filters in support of  $\mathcal{Q}$ .

In the context of peer grading, the first condition basically says that a person judging that a homework submission deserves a grade A is more likely if the true grade is A (in fact, more than  $\delta$  so), than if the true grade is something else, which is quite intuitive and can be safely assumed to hold. Note that this is different from saying that

$$p_j(h_1|h_1) > p_j(h_2|h_1) + \delta,$$

which says, for example, that if the true grade is A, then it is more likely that a person thinks it is A than it is something else. In fact one can show that, this condition implies regularity (and hence is stronger than regularity) if  $|\mathcal{H}| = |\mathcal{S}| = 2$ .<sup>3</sup>

Also, in the case of peer grading, the true type of the answer may be drawn from a rich space of features like we have discussed earlier, e.g., handwriting, presentation etc. In such settings, the regularity assumption may be violated with respect to this entire space of types, since different students may be biased towards considering different features of an answer more important than others. For example one student may value presentation of the answer more than elegance of the solution or handwriting, while other may have different biases. However, in practice, each submission is typically evaluated separately with respect to each of these features. It is reasonable to assume that the regularity assumption is well justified with respect to each of these individual features. Now we will consider the class of generating models  $\mathcal{C}_{reg}$ .

**Definition 5.3.** Suppose that  $|\mathcal{S}| = 2$ .  $\mathcal{C}_{reg}$  is defined to be the class of generating models  $(P_X, \mathcal{Q})$  that satisfy the following set of assumptions.

1. Each generating model  $(P_X, \mathcal{Q}) \in \mathcal{C}_{reg}$  is  $\delta_0$ -regular for some  $\delta_0 > 0$ .
2. There is an  $\epsilon_0 > 0$  such that  $P_X(h) > \epsilon_0$  for each  $h \in \mathcal{H}$  for all generating models in  $\mathcal{C}_{reg}$ .

The proposed mechanism for this class is presented in Mechanism 2 (Het-OA). We then have the following result.

**Theorem 3.** Consider a sequence of populations  $\{\mathcal{M}(N)\}$  indexed by the number of objects  $N$  that satisfies  $|\mathcal{M}(N)^i| \geq 2$  and  $|\mathcal{W}(N)_j| \leq C$ , where  $C$  is finite. Then there is an  $N_0$  depending only on  $\delta_0$ ,  $\epsilon_0$  and  $K$  such that for all  $N > N_0$ , Mechanism 2 is strictly detail-free Bayes-Nash incentive compatible with respect to the class  $\mathcal{C}_{reg}$ .

---

<sup>3</sup>In fact, there are many situations where this stronger condition may not hold. A popular example is that many people do not know that the capital of the US state of Illinois is Springfield, and not Chicago, as is commonly assumed. That is, given that the capital is Springfield, it is not true that a majority of the population thinks that it is Springfield. But it is reasonable to assume that more people think that the capital is Springfield if the capital was actually Springfield (which it is) than if the capital was something else, say Chicago. This is precisely the regularity assumption.

---

**Mechanism 2:** Het-OA: Heterogeneous population with binary evaluations and regular filters. Assumes that  $|\mathcal{M}^i| \geq 2$ .

---

The observations of all the people for the different objects are solicited. Let these be denoted by  $\{q_j^i\}$ , where every  $q_j^i \in \mathcal{S}$ . A person  $j$ 's payment is computed as follows:

- Form the largest set possible of distinct persons different from  $j$  such that each person has evaluated a different object from everyone else. Denote this set by  $\mathcal{U}_j$  and the set of different objects evaluated by this set by  $\mathcal{A}_j$ . For each  $i$  in  $\mathcal{A}_j$ , let  $j(i)$  denote the person in  $\mathcal{U}_j$  that has evaluated  $i$ . Next, for each of the two evaluations  $s_1$  and  $s_2$ , and for  $i \in \mathcal{A}_j$ , define

$$f_j^i(s) = \mathbf{1}_{\{q_{j(i)}^i = s\}}.$$

Then compute

$$\bar{f}_j(s) = \frac{1}{|\mathcal{A}_j|} \sum_{i \in \mathcal{A}_j} f_{j(i)}^i(s).$$

- For the evaluations  $s_1$  and  $s_2$ , fix payments  $r_j(s_1)$  and  $r_j(s_2)$  defined as

$$r_j(s) = \begin{cases} \frac{K}{\bar{f}_j(s)} & \text{if } \bar{f}_j(s) \neq 0, \\ 0 & \text{if } \bar{f}_j(s) = 0, \end{cases}$$

where  $K > 0$  is any positive constant.

- For computing person  $j$ 's payment for evaluating object  $o \in \mathcal{W}_j$ , choose another person  $j'$  who has evaluated the same object  $o$ . If their reports match, i.e., if  $q_j^i = q_{j'}^i = s$ , then the person  $j$  gets a reward of  $r_j(s)$ . If the reports do not match, then  $j$  gets 0 payment for the evaluation of that object.
- 

The high-level idea of the proof is the following. First,  $|\mathcal{W}(N)_j| \leq C$ , where  $C$  is finite, means that each person evaluates only a finite number of objects. This together with the fact that  $|\mathcal{M}(N)^i| \geq 2$ , i.e., each object is evaluated by somebody, means that there are at least  $\lfloor N/C \rfloor$  evaluations of distinct objects that are made by distinct persons, and thus  $|\mathcal{A}_j| \geq \lfloor N/C \rfloor - 1 = O(N)$ . Hence by the law of large numbers, for an agent  $j$ ,  $\bar{f}_j(s)$  will converge to its mean as the number of objects grows, which is the probability that an arbitrarily chosen person in the population will report a signal  $s$  for an arbitrary object. This probability is given by

$$P(Y = s) = \sum_{h \in \mathcal{H}} P_X(h) E(p_j(s|h)) = \sum_{h \in \mathcal{H}} P_X(h) p(s|h) > \epsilon_0 \delta_0.$$

By arguments similar to those used in Theorem 1, one can show that  $E(r_j(s))$  converges to  $K/P(Y = s)$ . Now suppose that an agent makes the evaluation  $s_1$ . Then our assumptions on the generating model, in particular the  $\delta_0$ -regularity condition ensures that  $P(Y_{j'}^i = s_1 | Y_j^i = s_1) - P(Y = s_1) > 0$ , and hence  $P(Y_{j'}^i = s_2 | Y_j^i = s_1) - P(Y = s_2) < 0$ . In words, *conditional on making a particular evaluation, the probability that a randomly picked agent that has evaluated the same object makes the same evaluation increases relative to the unconditional probability of an arbitrary agent making that evaluation for an arbitrary object.* Thus

$$K \frac{P(Y_{j'}^i = s_1 | Y_j^i = s_1)}{P(Y = s_1)} > K \frac{P(Y_{j'}^i = s_2 | Y_j^i = s_1)}{P(Y = s_2)}.$$

For a large  $N$ , the  $LHS$  is approximately the expected payment the agent receives if she reports  $s_1$  and  $RHS$  is approximately the expected payment if she reports  $s_2$  (with the approximation error decaying in  $N$ ). Further the regularity assumption implies that the gap in this inequality is actually positive (this gap depends on  $\delta_0$  and  $\epsilon_0$ ). Hence the mechanism is truthful for a large enough  $N$ .

[RFJ16] propose a similar output agreement mechanism where reward for matching is scaled by the prior probability of the evaluation, that is shown to be truthful in the non-binary setting assuming the condition:

$$\arg \max_{s' \in \mathcal{S}} \frac{P(Y_{j'}^i = s' | Y_j^i = s)}{P(Y = s')} = s.$$

Clearly, under this condition, our mechanism is also truthful in the non-binary setting. But in the non-binary setting, it is unclear if this condition is true under reasonable assumptions, unlike regularity in the binary setting. In fact, it is not even clear if this condition is true under reasonable assumptions in the homogeneous population setting.

*Proof of Theorem 3.* By arguments similar to the ones presented in the proof of Theorem 1, we can show that

$$E(r_j(s)) = E(\mathbf{1}_{\{\bar{f}_j(s) \neq 0\}} \frac{K}{\bar{f}_j(s)}) = \frac{K}{P(Y = s)} + m(N)$$

for  $k = 1, 2$ , where  $|m(N)| \leq \sigma(N) = o(1)$ , where  $\sigma(N)$  depends only on  $\epsilon_0$ ,  $\delta_0$  and  $K$ , and not on the particular choice of generating model in  $\mathcal{C}_{reg}$ . Next we have

$$P(X^i = h_l | Y_j^i = s_1) = \frac{P_X(h_l) p_j(s_1 | h_l)}{\sum_{h_l} P_X(h_l) p_j(s_1 | h_l)} = P_X(h_l) t(s_1, h_l),$$

where we have defined  $t(s_1, h_l) \triangleq \frac{p_j(s_1 | h_l)}{\sum_{h_l} P_X(h_l) p_j(s_1 | h_l)}$ . Because of strict regularity of the filters, we have that

$$t(s_1, h_1) > t(s_1, h_2) > \dots > t(s_1, h_L)$$

and further  $t(s_1, h_1) > 1$ . Now from the point of view of person  $j$ , the distribution of the report of a randomly chosen person  $j'$  who has evaluated the same object is

$$P(Y_{j'}^i = s_1 | Y_j^i = s_1) = \sum_{h_l} P(X^i = h_l | Y_j^i = s_1) p(s_1 | h_l) = \sum_{h_l} P_X(h_l) t(s_1, h_l) p(s_1 | h_l),$$

and  $P(Y_{j'}^i = s_2 | Y_j^i = s_1) = 1 - P(Y_{j'}^i = s_1 | Y_j^i = s_1)$ . This holds because  $Y_j^i$  for different persons  $j$  and  $j'$  are conditionally independent given the type  $X^i$ . Next we have

$$P(Y_{j'}^i = s_1 | Y_j^i = s_1) - P(Y = s_1) = \sum_{h_l} p(s_1 | h_l) \left( P_X(h_l) t(s_1, h_l) - P_X(h_l) \right).$$

We want to show that this quantity is positive and bounded away from 0, i.e., if a person has an evaluation  $s_1$  for an object, then the posterior probability that another person also has the same evaluation for that object strictly increases relative to the prior. Now since  $t(s_1, h_1) > t(s_1, h_2) > \dots > t(s_1, h_L)$  and  $t(s_1, h_1) > 1$ , since  $\sum_{h_l} P_X(h_l) (t(s_1, h_l) - 1) = 0$ , we cannot have  $t(s_1, h_L) > 1$  for all  $l$ , and hence there must be some  $l^*$  such that  $t(s_1, h_l) \geq 1$  for all  $l \leq l^*$  while  $t(s_1, h_l) < 1$  for all  $l > l^*$ . Then we have that

$$\sum_{h_l; l \leq l^*} P_X(h_l) (t(s_1, h_l) - 1) > 0 \tag{9}$$

and

$$\sum_{h_l; l \leq l^*} P_X(h_l)(t(s_1, h_l) - 1) = - \sum_{h_l; l > l^*} P_X(h_l)(t(s_1, h_l) - 1). \quad (10)$$

In fact, we can prove the following lower bound:

$$\begin{aligned} \sum_{h_l; l \leq l^*} P_X(h_l)(t(s_1, h_l) - 1) &\geq P_X(h_1)(t(s_1, h_1) - 1) \\ &= \frac{P_X(h_1)p_j(s_1|h_1)}{\sum_{h_l} P_X(h_l)p_j(s_1|h_l)} - P_X(h_1) \\ &> \frac{P_X(h_1)p_j(s_1|h_1)}{P_X(h_1)p_j(s_1|h_l) + (1 - P_X(h_1))p_j(s_1|h_2)} - P_X(h_1) \\ &> \frac{P_X(h_1)}{P_X(h_1)p_j(s_1|h_l) + (1 - P_X(h_1))(p_j(s_1|h_1) - \delta_0)} - P_X(h_1) \\ &= \frac{P_X(h_1)(1 - P_X(h_1))\delta_0}{p_j(s_1|h_l) - (1 - P_X(h_1))\delta_0} \\ &> P_X(h_1)(1 - P_X(h_1))\delta_0 \\ &> \epsilon_0(1 - \epsilon_0)\delta_0 \end{aligned} \quad (11)$$

Now we have:

$$\begin{aligned} &P(Y_{j'}^i = s_1 | Y_j^i = s_1) - P(Y = s_1) \\ &= \sum_{h_l; l \leq l^*} p(s_1|h_l)P_X(h_l)(t(s_1, h_l) - 1) + \sum_{h_l; l > l^*} p(s_1|h_l)P_X(h_l)(t(s_1, h_l) - 1) \\ &\geq p(s_1|h_{l^*}) \sum_{h_l; l \leq l^*} P_X(h_l)(t(s_1, h_l) - 1) + p(s_1|h_{l^*+1}) \sum_{h_l; l > l^*} P_X(h_l)(t(s_1, h_l) - 1) \\ &= \left( p(s_1|h_{l^*}) - p(s_1|h_{l^*+1}) \right) \sum_{h_l; l \leq l^*} P_X(h_l)(t(s_1, h_l) - 1) \\ &> \delta_0^2 \epsilon_0(1 - \epsilon_0) \triangleq \omega_0. \end{aligned}$$

The first inequality follows from the fact that  $P_X(h_l)t(s_1, h_l) - P_X(h_l) \geq 0 (< 0)$  for  $l \leq l^* (> l^*)$  and that  $p(s_1|h_1) > p(s_1|h_2) > \dots > p(s_1|h_L)$  by the strict regularity of the ensemble filter. The second equality follows from (11) and (10). It follows that

$$P(Y_{j'}^i = s_2 | Y_j^i = s_1) - P(Y = s_2) < -\omega_0.$$

Hence the expected payoff if a person  $j$  reports  $s_1$ , if he observes  $s_1$  is

$$\begin{aligned} \phi_{s_1|s_1} &= P(Y_{j'}^i = s_1 | Y_j^i = s_1)E(r_j(s_1)) \\ &> (P(Y = s_1) + \omega_0)E(r_j(s_1)) \\ &> K + \frac{\omega_0 K}{P(Y = s_1)} - |m(N)|(1 + \omega_0) \\ &> K + \omega_0 K - |\sigma(N)|(1 + \omega_0), \end{aligned}$$

and the expected payoff if the person reports  $s_2$  instead is

$$\begin{aligned}
\phi_{s_2|s_1} &= P(Y_{j'}^i = s_2 | Y_j^i = s_1) E(r_j(s_2)) \\
&< (P(Y = s_2) - \omega_0) E(r_j(s_2)) \\
&= K - \frac{\omega_0 K}{P(Y = s_2)} + |m(N)|(1 + \omega_0) \\
&< K - \omega_0 K + |\sigma(N)|(1 + \omega_0).
\end{aligned}$$

Thus for  $N > N_0$ , where  $N_0$  depends only on  $\epsilon_0$ ,  $\delta_0$  and  $K$ , being truthful maximizes the expected payment.  $\square$

## 5.2 Remarks

**A Mechanism for any  $N$ :** It may be tempting to think that one can achieve incentive-compatibility in expectation if instead of using  $\bar{f}_j(s_k)$  as an asymptotically accurate estimate of  $P(Y = s_k)$ , one can simply draw a person  $j'$  from just one population, say  $i$ , and use  $f_j^i(s_k) = 1_{\{Y_{j'}^i = s_k\}}$  to compute the scores  $r_j(s_k)$ . But although  $E(f_j^i(s_k)) = P(Y = s_k)$ , clearly  $E(\frac{1}{f_j^i(s_k)}) \neq \frac{1}{P(Y = s_k)}$ . Hence incentive-compatibility does not hold in general. Nevertheless, if we can depart from the format of an output agreement mechanism, then one can retrieve incentive-compatibility for any finite  $N$  using the following "additive" mechanism (again assuming binary evaluations and regularity of filters).

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**Mechanism 3:** Het-additive: Heterogeneous population with binary evaluations and regular filters

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- The observations of all the people for the different objects are solicited. Let these be denoted by  $\{q_j^i\}$ .
- For each person  $j$  in population  $\mathcal{M}^i$ , pick a person  $j'$  from the same population  $\mathcal{M}^i$ , and another person  $j''$  from a different population  $\mathcal{M}^{i'}$ . The reward of person  $j$  is:

$$K [\mathbf{1}_{\{Y_j^i = Y_{j'}^i\}} + \mathbf{1}_{\{Y_j^i \neq Y_{j''}^i\}}],$$

where  $K > 0$  is an arbitrarily chosen positive constant.

---

To see that this mechanism is truthful, first note that the reward to person  $j$ , can equivalently written as

$$K + K [\mathbf{1}_{\{Y_j^i = Y_{j'}^i\}} - \mathbf{1}_{\{Y_j^i = Y_{j''}^i\}}].$$

Ignoring the constant reward  $K$ , the expected additional payment in the mechanism if she tells the truth, if she makes an evaluation  $s_1$ , is  $K [P(Y_{j'}^i = s_1 | Y_j^i = s_1) - P(Y = s_1)]$ . But by the regularity assumption, as mentioned above, we have shown in the proof of Theorem 3 that  $P(Y_{j'}^i = s_1 | Y_j^i = s_1) - P(Y = s_1) > 0$ , and hence  $P(Y_{j'}^i = s_2 | Y_j^i = s_1) - P(Y = s_2) < 0$ . Thus the mechanism is strictly detail-free Bayes-Nash incentive compatible. [SAFP16] define the following condition for non-binary signals in the heterogeneous population setting:  $P(Y_{j'}^i = s | Y_j^i = s) - P(Y = s) > 0$  and  $P(Y_{j'}^i = s' | Y_j^i = s) - P(Y = s') < 0$  for every  $s' \neq s$ , and they present a mechanism that is truthful under this condition. It is easy to see that if a generating model satisfies this property in our setting, then the Het-additive mechanism is strictly detail-free Bayes-Nash incentive compatible

with respect to this generating model. But again it is not clear if there are practically reasonable structural assumptions on the generating model that imply this property.

**Using Mechanism 2 for the homogeneous case with binary evaluations:** Mechanism 2 is asymptotically truthful for the case of a homogeneous population for binary-choice evaluation tasks under the appropriate regularity assumptions. Note that in that case since everyone has the same filter, the 0-regularity condition is immediately satisfied. Also note that Mechanism 2 is not truthful in general in the homogeneous setting for non-binary evaluation tasks.

**Other equilibria:** Again in this case there are other equilibria of the class where everybody reports an independent sampling from some common distribution, but in expectation they give a lower payoff than the truthful equilibrium. Similar to what we saw in the homogeneous setting, asymptotically (in  $N$ ) the expected reward is  $K$  under this equilibrium. But it is clear from the proof of Theorem 3 that if the filters are  $\delta_0$ -regular, the expected reward under the truthful equilibrium asymptotically is strictly greater than  $K$ .

## 6 Experiments

Motivated by the significant emphasis on simplicity of mechanisms in the community [Rub15, Li15, Sim15], we performed experiments on the Amazon Mechanical Turk crowdsourcing platform to test the ‘understandability’ or ‘simplicity’ of the structure of our mechanisms as compared to prior art. By simplicity, we roughly mean the ability of the mechanism to induce optimal behavior in agents who are presented with it. Indeed, “simplicity” of the mechanism bears intrinsic value and several applications demand an understanding of the underlying mechanism: for instance, students wish to know how their grades are computed, voters wish to know what electoral system is used etc. In what follows, we first describe the experimental setup. Alongside, we also detail the various challenges associated to such a study on Amazon Mechanical Turk and our approaches in overcoming these challenges, which may be of independent interest. Subsequently, we detail the outcomes of the experiment.

### 6.1 Experimental setup and challenges

There are several challenges in designing experiments to study such a characteristic of these mechanisms. Tasks on MTurk are usually objectively verifiable and include image recognition, transcription and matching tasks [RYZ<sup>+</sup>10]. As a consequence, workers may expect that their work will be verified and rewarded according to their objective accuracy [SZ15]. This feature introduces a significant bias for testing incentive compatible mechanisms, as workers have a tendency to report the truth [GMCA14]. This effect is amplified when the worker himself or herself is the “agent” in the mechanism. Moreover, in order to garner interest from worker, and thereby obtain quality data for the experiment, it is quite essential to make the task interesting [KSV11, BKG11].

In order to address these issues, we present the worker with a game-like setting, where he or she interacts with the experiment via a character (see Figure 2(a) for an example). The goal of the worker is to maximize the payment obtained by the character. We explicitly account for the worker’s bias through the introduction of an ‘inverted mechanism’ for each of our candidate mechanisms, that results from a trivial modification in which all the payments to the agent in the original mechanism are presented as penalties to the worker (over a base reward that ensures that the net payments are non-negative). Hence in these inverted mechanisms, truth-telling is no longer an optimal strategy:

in the binary setting, lying is an optimal strategy. Any worker is shown either the normal or the inverted setting, chosen uniformly at random. We also perform an additional randomization of the signal (“A” or “B”) observed by the character in the experiment. The game-based setting is further made interesting via illustrations, examples, and simple-to-understand language for the mechanism descriptions.

We compared the HET-OA mechanism of the present paper with the mechanism of [RF15] (which we will denote as RF15).<sup>4</sup>

In more detail, the experiment was structured as follows:

- Upon visiting the platform, workers are presented with a fictional character “Sam”, whom they will help. Sam’s observation (ground truth) is provided to the worker, as well as a question eliciting that observation.
- In the setting, Sam is the grader for an essay in high school, and the principal of the school has announced a reward/penalty according to the quality of grades given by each student. The goal of the worker is to help Sam choose the grade (out of  $\{A, B\}$ ) that will maximize Sam’s reward (or minimize Sam’s penalty).
- Each worker is now presented with one of the eight settings (we sample, uniformly at random, which mechanism to present, whether to invert it and which private signal to show) at random and asked whether Sam should answer the question truthfully in order to maximize his payoff assuming everyone else answers truthfully. Each worker only interacts with one setting to minimize possible bias.
- In addition to providing a (binary) recommendation for Sam, workers are also asked to provide a brief justification.

Note that the worker is explained the mechanism but is not told whether the mechanism is attempting to induce the truth or a lie from the worker. Each worker is offered a fixed payment of 30 cents, and contingent on giving the correct answer and a reasonable explanation, an additional bonus of the same amount.

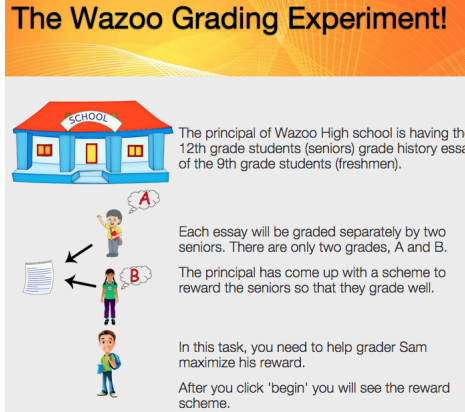
## 6.2 Mechanism Descriptions

Since our goal was to gauge the ‘understandability’ of our mechanisms in the general population, we did not wish to restrict our experiment to agents with a background in mathematics or quantitative sciences. This consequently meant that we had to explain both mechanisms without any mathematical expressions, beyond simple operations like addition and multiplication. We presented our mechanism by means of a lookup table and presented the RF mechanism (where there is no intuitive use for such a table) with a sequence of 3 steps of simple addition. Both mechanisms were accompanied with an example scenario where the payment computation steps are shown along with the final payment.

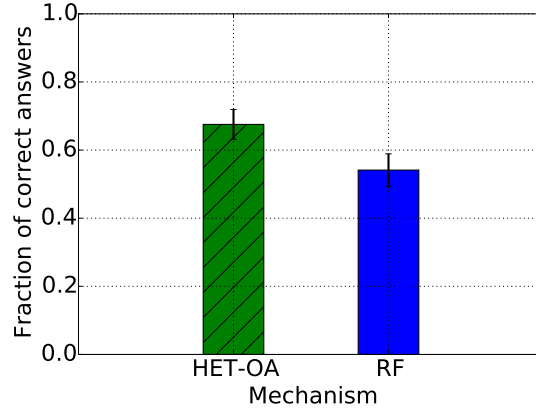
In the appendix, we present the exact text of the description of the mechanisms given to the workers along with the belief structure of the fictional character. This allowed us to quickly categorize their answers as correct or incorrect for our analysis.

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<sup>4</sup>The mechanism descriptions, data and a link to the actual survey are available at <https://eecs.berkeley.edu/~nihar/data/nogold>



(a) Illustration of the interface presented to the workers.



(b) Fraction of workers who answered correctly (that is, according to the requirements of that mechanism).

Figure 2: The platform and the results from experiments on Amazon Mechanical Turk comparing the RF mechanism [RF15] and the HET-OA mechanism proposed in the present paper. The difference between the two mechanisms is statistically significant ( $p < 0.05$ ).

### 6.3 Results and analysis

Figure 2(b) plots the results from the experiments, showing the fraction of workers who answered correctly (that is, according to what the mechanism was designed to incentivize). We obtained a total of 223 workers, with 114 workers being shown our mechanism and 109 workers shown the RF mechanism.

In more detail, the number of workers ( $n$ ) and the fraction of correct responses ( $\mu$ ) and the standard error of the mean ( $\epsilon$ ) associated to each sub-class of the tasks is as follows:

Mechanism	$n$	$\mu$	$\epsilon$
HET-OA	61	0.803	0.051
Inverted HET-OA	53	0.528	0.069
RF	60	0.500	0.065
Inverted RF	49	0.592	0.070

Observe from the table that on average, the workers' responses are more accurate under HET-OA than RF. We employ the standard two-sample t-test [Sta00] between the two sets of data in order to investigate whether this difference is statistically significant or not. The t-test is used to test whether the samples in two sets of data are drawn from an underlying distribution with identical means (the null hypothesis) or not (the alternative hypothesis). In our case, the mean is simply the mean of a Bernoulli distribution that dictates the likelihood of a worker's response being correct. The t-test reveals that HET-OA mechanism elicited a statistically significant number of more correct responses (with a p-value of 0.02) as compared to RF, indicating that the mechanism proposed in this paper was more understandable to the workers as compared to the RF mechanism.

All in all, these experiments indicate, in a statistically significant manner, that output agreement-type mechanisms with popularity-scaled rewards such as those proposed in the present paper are more intuitive and understandable, and hence more successful in inducing truthful behavior.



## 7 Conclusion

We presented mechanisms for obtaining truthful reports with minimal elicitation. Our mechanisms support the setting where agents are assumed to be homogeneous, and also support heterogeneous workers when questions are of binary-choice format. The mechanisms rely on the existence of many questions, a feature commonly encountered in the settings of crowd-sourcing and peer-grading. We experimentally tested our mechanisms and found ( $p = 0.02$ ) that they are more understandable and better at inducing truthful behavior than the current state of the art. Interestingly, our mechanisms are built under a novel framework that is a significant departure from the traditional setup of proper scoring rules.

Our broader objective is to construct mechanisms that incentivize truthful reports in the absence of any ‘gold standard’ questions, that are also viable in practice. The results of this paper take a significant step in this direction. A question left open in this manuscript is that of the general  $|\mathcal{S}| > 2$  setting with a heterogeneous population. It would be interesting to find the right notion of "regularity" of the agents' filters (that is weaker than the condition in [SAFP16] or in [RFJ16]) that would allow one to design a strictly detail-free Bayes-Nash incentive compatible mechanism in that case.

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## 8 Appendix

### 8.1 Mechanism Descriptions

These descriptions pertain to the non-inverted case.

**HET-OA Mechanism Description** Suppose that Lisa is grading the same essay as Sam. Let’s see what reward Sam gets. First, he gets \$1 as a starting reward. Then he may also get a bonus reward. How much? That depends. If he and Lisa give different grades (e.g., Sam gives an A and Lisa gives a B), then Sam gets no bonus. However, if he and Lisa give the same grade, then he will get a bonus that depends on how popular the grade is overall. Higher the popularity, lower the bonus.

In general, if  $x\%$  of the class got an A and  $y\%$  got a B (of course,  $x+y$  must add up to 100) then Sam’s bonus for the different possibilities is given in the table below.

	Lisa gives A	Lisa gives B
Sam gives A	$100/x$	0
Sam gives B	0	$100/y$

For example, if 40% of the grades were A and 60% B, and if Lisa and Sam both give A, then Sam gets a bonus of \$2.5, but if both give a B, then their bonus is only \$1.67 since B is more popular.

**RF Mechanism Description** Since every essay is graded by two people, we will call those two students partners. Suppose that Lisa is Sam’s partner. First, for every other essay, one of the two graders that have evaluated it is chosen to form a collection of graders. If all the grades given by this collection do not have 2 As and 2 Bs, then Sam does not get any reward and the scheme ends.

If they have 2 As and 2 Bs, then choose two graders from this collection who gave the same grade to their essays as Sam. Suppose these graders are Bob and Mike. Let Bob’s partner be Alice and let Mike’s partner be Nicole.

Sam is then rewarded as follows:

- He gets a starting reward of \$0.50.
- He gets a bonus of \$1 if Lisa's grade is same as Alice's.
- But then he pays a penalty of \$0.5 if Alice's grade is same as Nicole's.

For instance, if Sam gives an B, while Lisa, Nicole and Alice give an A, Sam would receive a total reward of  $0.5 + 1 - 0.5 = \$1$ .

**Belief Structure** In addition to the description of the mechanism, workers were also presented with three sentences pertaining to the beliefs of the fictional character Sam:

1. Sam believes that 20% of the essays will get an A and 80% will get a B if everyone grades honestly.
2. Suppose that Sam thinks that his essay deserves an A, and he thinks that Lisa will also give it an A with a 40% chance.

Assuming that every other grader is going to grade honestly, what grade should Sam report to maximize his reward?